Reducing Boolean Expressions Using Logic Fundamentals

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Introduction

The intent of this document is to help you know various laws in logic fundamentals and show their application by reducing few boolean expressions.

First, we will understand following logic fundamental laws:

- Complementation Law
- Commutative Law
- Associative Law
- Distributive Law
- Or Law
- And Law

Then we will apply these laws to reduce boolean expressions.

Logic Fundamentals

Below are listed various laws that will later be used to reduce the boolean expressions:

Complementation Laws

1. 0’ = 1
2. 1’ = 0
3. A’ = A

Commutative Laws

1. A+B = B+A
2. A.B = B.A

Associative Laws

1. A+(B+C) = (A+B)+C
2. A.(B.C) = (A.B).C

Distributive Laws

1. A.(B+C) = AB + AC
2. A+(B.C) = (A+B).(A+C)
3. A+ A’B = A+B

OR Laws

1. 0+0 = 0
2. 0+1 = 1
3. 1+0 = 1
4. 1+1 = 1
5. A+0 = A
6. A+1 = A
7. A+A = A
8. A+A’ = 1

AND Laws

1. 0.0 = 0
2. 0.1 = 0
3. 1.0 = 1
4. 1.1 = 1
5. A.0 = 0
6. A.1 = A
7. A.A = A
8. A.A’ = 0

Reducing Boolean Expressions

Below are listed various examples to show how fundamental laws explained above can be used to reduce boolean expressions.

Q1. Prove that A + AB = A

A. A + AB = A . (1+B)
= A . 1 [OR Law 6: 1+B = 1]
= A

Q2. Prove that (A+B) (A+C) = A + BC

A. (A+B) (A+C) = A.A + A.C + B.A + B.C
= A + AC + BA + BC [AND Law 7: A.A = A]
= A.(1+C) + BA + BC
= A + BA + BC [OR Law 6: 1+C = 1]
= A (1+B) + BC
= A + BC [OR Law 6: 1+B = 1]

Q3. Prove that A + A'B = A + B

A. A + A'B = A.1 + A'B
= A.(1+B) + A'B [OR Law 6: 1+B = 1]
= A + AB + A'B
= A + (A+A').B
= A + 1.B [OR Law 8: A+A' = 1]
= A + B

Q4. Prove that A' + AB = A' + B

A. A' + AB = A'.1 + AB
= A'.(1+B) + AB [OR Law 6: 1+B = 1]
= A' + A'B + AB
= A' + (A'+A).B
= A' + 1.B [OR Law 8: A+A' = 1]
= A' + B

Q5. Prove that A . (A+B) = A

A. A . (A+B) = A.A + A.B
= A + AB
= A.(1+B) [OR Law 6: 1+B = 1]
= A.1
= A

Q6. Prove that (A+B') . B = AB

A. (A+B') . B = AB + B'B
= AB + 0 [AND Law 8: B'B = 0]
= AB

Q7. Prove that AB' + A'B = (A+B) (AB)'

A. AB' + A'B = A.A' + A.B' + B.A' + B.B' [Adding AA' & BB']
= A.A' + B.A' + A.B' + B.B'
= (A+B) A' + (A+B) B'
= (A+B) (A'+B')
= (A+B) (AB)' [De Morgan's Law – A'+B' = (AB)']

Q8. Prove that AB + A'B' = [(AB)' (A'B')']' = [(A'+B') (A+B)]'

A. AB + A'B' = [(AB + A'B')']'
= [(AB)' (A'B')']' [De Morgan's Law – A'+B' = (AB)']
= [(A'+B') (A+B)]' [De Morgan's Law – (AB)' = A'+B']

Q9. Prove that AC + A'BC = AC + BC

A. AC + A'BC = (A + A'B) . C
= (A+B) . C [Distributive Law 3: A+A'B = A+B]
= AC + BC

Q10. Prove that AB + A'C = (A+C) (A'+B)

A. AB + A'C = AB.(1+C) + A'C [OR Law 6: 1+C = 1]
= AB + ABC + A'C
= AB + (AB+A') C
= AB + (A'+B) C [Distributive Law 3: A'+AB = A'+B]
= AB + AA' + (A'+B)C
= A (A' + B) + (A'+B)C
= (A+C) (A'+B)

Q11. Prove that A’B’C + (A+B+C’)’ + A’B’C’D = A’B’(C+D)

A. A’B’C + (A+B+C’)’ + A’B’C’D = A’B’C + (A+B)’.C + A’B’C’D [De Morgan’s Law – (A+B)’ = A’B’]
= A’B’C + A’.B’.C + A’B’C’D [De Morgan’s Law – (A+B)’ = A’B’]
= A’B’C + AB’C’D [OR Law 7: A+A = A]
= A’B’(C+D)

Q12. Prove that (B+BC)(B+B’C)(B+D) = BC

A. (B+BC)(B+B’C)(B+D) = (B(1+C)) ((B+B’)C) (B+D)
= (B.1) (1.C) (B+D) [OR Law 6: 1+C = 1 & OR Law 8: B+B’ = 1]
= BC (B+D)
= BC.B + BC.D
= (B.B) C + BCD [Associate Law 2: A.(B.C) = (A.B).C]]
= BC + BCD
= BC (1+D)
= BC

Q13. Prove that (B’D + A’BC’ + ACD + A’BC)’ = ABC’ + AD’ + B’D’

A. (B’D + A’BC’ + ACD + A’BC)’ = (B’D + A’BC’ + A’BC + ACD)’ [Associate Law 1: A+(B+C) = (A+B)+C]
= (B’D + A’B(C’+C) + ACD)’
= (B’D + A’B + ACD)’ [OR Law 8: C+C’ = 1]
= (B’D + ACD + A’B)’ [Associate Law 1: A+(B+C) =A+B)+C]
= ((B’+AC)D + A’B)’
= ((B’+AC)D)’ . (A’B)’ [De Morgan’s Law – (A+B)’ = A’B’]
= ((B’+AC)’ + D’) . (A + B’) [De Morgan’s Law – (AB)’ = A’+B’]
= ((B.(AC)’ +D’) . (A+B’) [De Morgan’s Law – (A+B)’ = A’B’]
= (B.(A’+C’) + D’) . (A+B’) [De Morgan’s Law – (AB)’ = A’+B’]
= (A’B + BC’ + D’) (A+B’)
= A’B.A + A’B.B’ + BC’.A + BC’.B’ + D’A + D’B’
= 0 + 0 + ABC’ + 0 + AD’ + B’D’ [AND Law 8: A’A = 0]
= ABC’ + AD’ + B’D’

Q14. Prove that ((AB’ (C+BD) + A’B’)C)’ = B+C’

A. ((AB’ (C+BD) + A’B’)C)’ = ((AB’.C + AB’.BD) + A’B’)C)’
= ((AB’.C + 0) + A’B’)C)’ [AND Law 8: A’A = 0]
= ((AB’C + A’B’)C)’
= (AB’C.C + A’B’.C)’
= (AB’C + A’B’C)’ [AND Law 7: AA = A]
= ((A+ A’)B’C)’

= (B'C)' [OR Law 8: A+A' = 1]
= B + C' [De Morgan's Law – (AB)' = A'+B']

Q15. Prove that ((A'+B+C+D)' (AB'C'D)')' = A' + B +C + D

A. ((A'+B+C+D)' (AB'C'D)')' = ((A'+B+C+D)')' + ((AB'C'D)')' [De Morgan's Law – (AB)' = A'+B']
= A'+B+C+D + AB'C'D [(A')' = A]
= A'+B+C+D(1 + AB'C')
= A'+B+C+D [OR Law 6: 1+C = 1]

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